Effect of throughflow and Coriolis force on Bénard convection in micropolar fluids

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Abstract *The effect of throughflow and Coriolis force on convective instabilities in micropolar fluid layer heated from below for free-free, isothermal and micro-rotation free boundaries is investigated. Calculations are made using a lower order Galerkin approximation to solve the eigenvalue problem for stationary instability. It is observed that both stabilizing and destabilizing factors due to constant vertical throughflow can be enhanced by rotation.*

1. Introduction

The determination of the criterion for the onset of convection in a horizontal fluid layer heated uniformly from below is a classical problem associated with Bénard (1900) and Rayleigh (1916). Convection in viscous fluids has been extensively studied by Chandrasekhar (1961). The experiments of Sutton (1951), Degraff and Derheld (1953), Suberman (1959) and Spangenberg and Rowland (1961) reveal that the actual value of the critical Rayleigh number at which convection sets in depends on the rate of heating, and the resulting fluid motion is columnar in structure rather than the usual two-dimensional rolls. Theoretical studies of the onset of convection with non-linear basic temperature profiles, when there is no rotation, were made by Currie (1967), Nield (1975), Rudraiah *et al.* (1980) and Rudraiah (1982). In the presence of rotation, Veronis (1966) studied the role of convection. The effect of vertical mass discharge, known as throughflow, on the Bénard convection has been studied by Wooding (1960), Sutton (1970), Homsy and Sherwood (1976), Jones and Persichetti (1986), Nield (1987), Rudraiah (1989), Shivakumara and Jayaram (1993), Shivakumara and Rudraiah (1997) and Siddheswar and Pranesh (1997).

The effect of throughflow on convective instabilities is of interest because of the possibility of controlling the convective instabilities by the adjustment of vertical mass discharge called throughflow. The *in situ* processing of energy resources such as coal, oil shale or geo-thermal energy often involves the throughflow in porous medium. Jones and Persichetti (1986) considered the problem of convective instabilities in packed beds with throughflow in view of its applications in the reaction zone in the catalytic reactor or an ion exchange column.

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In recent years, there have been several investigations dealing with the thermal instability of micropolar fluids proposed by Eringen (1966). His theory serves as a satisfactory model to describe the flow of colloidal, polymeric, real fluids with suspensions, liquid crystals and animal blood. The Bénard problem for these fluids was studied by Lakshmana Rao (1970), Ahmadi (1976), Datta and Sastry (1976), Lebon and Garcia (1981), Bhattacharya and Jena (1983), Sastry and Ramamohan Rao (1983), Bhattacharya and Abbas (1985), Payne and Straughan (1989) and Qin and Kaloni (1992).

The problem of convective instability of a micropolar fluid layer heated from below finds applications in the area of geophysics. For example, in understanding the phenomenon of rising volcanic liquid, with bubbles and convective process inside the earth's mantle.

The effect of throughflow is, in general, quite complex. Not only is the basic temperature profile altered, but in the perturbation equations contributions arise from the convection of temperature, velocity and hence an overall interaction due to these contributions arises.

To add to the unlimited range of practical applications of micropolar fluids, it seems natural to extend the study of the convective instabilities of an electrically conducting micropolar fluid to include the effects of throughflow and rotation.

The theoretical study of the combined effect of rotation and throughflow on the Bénard convection in micropolar fluids is the main objective of the paper in view of its importance in planetary and stellar systems, geophysical and bio-mechanical applications, notably in the use of centrifugal separation devices.

In general, the determination of the critical Rayleigh number for a given situation entails solving the eigenvalue problem by numerical means. The variational method employed by Chandrasekhar (1961) requires elaborate numerical computations. To minimize these elaborate numerical computations, the Galerkin method is adopted, which is well documented in Finlayson (1972).

2. Formulation of the problem

Consider a layer of micropolar fluid between two horizontal planes of finite depth d extending to infinity in the two horizontal directions x and y with vertical throughflow of constant magnitude w_0 in z-direction. The fluid layer is initially isothermal and at rest. The boundaries are free-free, isothermal and free from micro-rotation. The fluid is heated from below so that a negative temperature gradient is maintained in the upward direction as shown in the schematic diagram (Figure 1).

Following Datta and Sastry (1976) and Qin and Kaloni (1992), the governing equations of motion for micropolar fluids proposed by Eringen (1966) are

$$\nabla \cdot \overline{V} = 0 \tag{1}$$

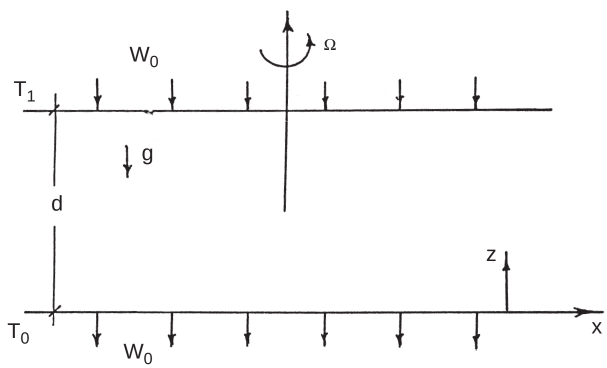

Figure 1.
Schematic diagram

$$\rho\left[\frac{\partial\overline{V}}{\partial t}+(\overline{V}\cdot\nabla)\overline{V}\right]-2\rho\overline{V}\times\overline{\Omega}=g\lambda\tau'\hat{K}-\nabla p+(\kappa+\mu)\nabla^2\overline{V}+\kappa\nabla\times\overline{\zeta} \tag{2}$$

$$\rho J\left[\frac{\partial\overline{\zeta}}{\partial t'}+(\overline{V}\cdot\nabla)\overline{\zeta}-\overline{\zeta}\times\overline{\Omega}\right]=(\alpha+\beta)\nabla\nabla\cdot\overline{\zeta}+\gamma\nabla^2\overline{\zeta}+\kappa[\nabla\times\overline{V}-2\overline{\zeta}] \tag{3}$$

$$\left[\frac{\partial}{\partial t'}+(\overline{V}\cdot\nabla)-K_c\nabla^2\right]\tau'=\phi\left[V_3-\frac{\delta}{\rho C_v}\psi_3\right] \tag{4}$$

with $\overline{V}(V_1, V_2, V_3)$ the velocity; $\overline{\zeta}(\zeta_1,\zeta_2,\zeta_3)$ the micro-rotation; ρ the density; p the pressure; J the micro-inertia; C_v the specific heat at constant volume; τ the temperature; $\mu,\kappa,\alpha,\beta,\gamma,\delta$ the material constants; ϕ the adverse temperature along the vertical direction; λ the coefficient of thermal expansion; t' time; K_c the thermal diffusivity and $\Omega(0,0,\Omega)$ the angular velocity.

In the quiescent state,

$$\overline{V}=w_0\hat{K},\overline{\Omega}=\Omega\hat{K},\overline{\theta}_b(z)=(t_1-T_0)\frac{[1-e^{w_0 z/k}]}{[1-e^{w_0\alpha/k}]} \tag{5}$$

where w_0 is the strength of the imposed constant vertical throughflow, $\hat{K}$ is the unit vector along z-direction and $\overline{\theta}_b(z)$ is the basic temperature distribution which is non-linear in nature due to throughflow.

Suppose that the quiescent state is slightly disturbed, then by taking the curl and curl curl of linearized version of equations (2) and (3) respectively and retaining the third component of the resulting equations, one obtains in non-dimensional form,

$$\left[\frac{\partial}{\partial t}+R_e\frac{\partial}{\partial z}-(1+R)\nabla^2\right]\nabla^2 w=R_a\nabla_1^2 T-E_k\frac{\partial\chi_1}{\partial z}+R\nabla^2\psi \tag{6}$$

$$P_r \frac{\partial T}{\partial t} - \nabla^2 T = w - \bar{\delta}\psi - P_e \frac{\partial T}{\partial z} - f(z)w \tag{7}$$

$$\frac{n_1 A}{R}\frac{\partial \psi}{\partial t} = \nabla^2 \psi - A\nabla^2 w - 2A\psi - \frac{n_1 A}{2R} E_k \chi_2 \tag{8}$$

$$\frac{\partial \chi_1}{\partial t} - E_k \frac{\partial w}{\partial z} + R_e \frac{\partial \chi_1}{\partial z} = (1+R)\nabla^2 \chi_1 - R\nabla_1^2 \zeta + R\frac{\partial \chi_2}{\partial z} \tag{9}$$

$$\frac{n_1 A}{R}\frac{\partial \zeta}{\partial t} = AB\frac{\partial}{\partial z}\left[\chi_2 + \frac{\partial \zeta}{\partial z}\right] + \nabla^2 \zeta + A[\chi_1 - 2\zeta] \tag{10}$$

$$\frac{n_1 A}{R}\frac{\partial \chi_2}{\partial t} - \frac{n_1 A}{2R} E_k \psi = AB\nabla_1^2\left[\chi_2 + \frac{\partial \zeta}{\partial z}\right] + \nabla^2 \chi_2 - A\frac{\partial \chi_1}{\partial z} - 2A\chi_2 \tag{11}$$

where the non-dimensional quantities are given by,

$$T = \frac{\tau'}{\phi d}; t = \frac{\mu t'}{\rho d^2}; w = \frac{V_3 d}{K_c}; \zeta = \frac{\zeta_3 d^2}{K_c}; \overline{\omega} = \nabla \times \overline{V}; \overline{\psi} = \nabla \times \bar{\zeta};$$
$$n_1 = \frac{J}{d^2}; \xi = \frac{\partial \zeta_1}{\partial x} + \frac{\partial \zeta_2}{\partial y};$$

microrotation parameter, $R = \frac{\kappa}{\mu}$; couple stress parameters, $A = \frac{\kappa d^2}{\gamma}, B = \frac{(\alpha+\beta)}{\kappa d^2}$; $\psi = \frac{\psi_3 d^3}{K_c}$; $\chi_1 = \frac{\omega_3 d^3}{K_c}$; $\chi_2 = \frac{\xi d^3}{K_c}$ Ekman number, $E_k = \frac{2\rho\Omega d^2}{\mu}$; Prandtl number, $P_r = \frac{\mu}{\rho K_c}$; coupling parameter, $\bar{\delta} = \frac{\delta}{\rho C_v d^2}$;temperature Rayleigh number, $R_a = \frac{\rho g \lambda \phi d^4}{K_c \mu}$; Péclet number, $P_e = \frac{w_0 d}{K-c}$; Reynolds number, $R_e = \frac{w_0 \rho d}{\mu}$; $(x, y, z) = \frac{1}{d}(x_1, x_2, x_3)$; $\nabla^2 = \frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial z^2}$; $\nabla_1^2 = \nabla^2 = \frac{\partial^2}{\partial z^2}$; the non-dimensional basic temperature gradient, $f(z) = \frac{\partial}{\partial z}\left(\frac{\bar{\theta}_b}{\Delta t}\right) = -\frac{P_e e^{P_e z}}{e^{P_e} - 1}$, is non-linear in z due to throughflow.

Making the normal mode analysis as in Chandrasekhar (1961) and Falin Chen (1990),

$$[w, T, \psi, \chi_1, \zeta, \chi_2] = [w'(z,t)\Theta'(z,t), G'(z,t), x'(z,t), y'(z,t), z'(z,t)]e^{i(a_1 x + a_2 y)}$$

where a_1 and a_2 are wave numbers.

Using these, equations (6)-(11) become

$$\left\{\frac{\partial}{\partial t} + R_e \frac{\partial}{\partial z} - (1+R)\left[\frac{\partial^2}{\partial z^2} - a^2\right]\right\}\left[\frac{\partial^2}{\partial z^2} - a^2\right] w' = -a^2 R_a \Theta' - E_k \frac{\partial x'}{\partial z} + R\left[\frac{\partial^2}{\partial z^2} - a^2\right] G' \tag{12}$$

$$\left\{ P_r \frac{\partial}{\partial t} - \left[\frac{\partial^2}{\partial z^2} - a^2 \right] + P_e \frac{\partial}{\partial z} \right\} \Theta' = [1 - f(z)] w' - \bar{\delta} G' \quad (13)$$

$$\left\{ \frac{n_1 A}{R} \frac{\partial}{\partial t} - \left[\frac{\partial^2}{\partial z^2} - a^2 \right] + 2A \right\} G' = -\frac{n_1 A}{2R} E_k z' - A \left[\frac{\partial^2}{\partial z^2} - a^2 \right] w' \quad (14)$$

$$\left\{ \frac{\partial}{\partial t} + R_e \frac{\partial}{\partial z} - (1 + R) \left[\frac{\partial^2}{\partial z^2} - a^2 \right] \right\} x' = E_k \frac{\partial w'}{\partial z} + a^2 R y' + R \frac{\partial z'}{\partial z} \quad (15)$$

$$\left\{ \frac{n_1 A}{R} \frac{\partial}{\partial t} - AB \frac{\partial^2}{\partial z^2} - \left[\frac{\partial^2}{\partial z^2} - a^2 \right] + 2A \right\} y' = AB \frac{\partial z'}{\partial z} + Ax' \quad (16)$$

$$\left\{ \frac{n_1 A}{R} \frac{\partial}{\partial t} + a^2 AB - \left[\frac{\partial^2}{\partial z^2} - a^2 \right] + 2A \right\} z' = \frac{n_1 A}{2R} E_k G' - a^2 AB \frac{\partial y'}{dz} - A \frac{\partial x'}{dz} \quad (17)$$

where $a^2 = a_1^2 + a_2^2$.

3. Application of Galerkin method

Choose suitable trial functions for velocity, vorticity, temperature and micro-rotation perturbations, as discussed by Finlayson (1972), in the form

$$w'(z,t) = \sum \alpha_i^*(t) W_i(z); \quad \Theta'(z,t) = \sum \beta_i^*(t) \Theta_i(z); \quad G'(z,t) = \sum \gamma_i^*(t) G_i(z)$$
$$x'(z,t) = \sum \delta_i^*(t) X_i(z); \quad y'(z,t) = \sum \theta_i^*(t) Y_i(z); \quad z'(z,t) = \sum \phi_i^*(t) Z_i(z)$$

where $W_i(z), \Theta_i(z), G_i(z), X_i(z), Y_i(z)$ and $Z_i(z)$ are polynomials in z that depend on appropriate boundary conditions and $\alpha_i^*, \beta_i^*, \gamma_i^*, \delta_i^*, \theta_i^*$ and ϕ_i^* are the amplitudes.

After substituting these trial functions in equations (12)-(17), multiply the resulting equations with $W_j(z), \Theta_j(z), G_j(z), X_j(z), Y_j(z)$ and $Z_j(z)$ respectively. Integration of the resulting equations with respect to z between 0 and 1 yields,

$$U_{ji} \frac{d\alpha_i^*}{dt} = E_{ji} \alpha_i^* + a^2 R_a O_{ji} \beta_i^* + R(Y5)_{ji} \gamma_i^* + E_k (Y6)_{ji} \delta_i^* \quad (18)$$

$$P_r H_{ji} \frac{d\beta_i^*}{dt} = I_{ji} \alpha_i^* + J_{ji}^* \beta_i^* + \bar{\delta} O_{ji} \gamma_i^* \quad (19)$$

$$\frac{n_1A}{R}(Y7)_{ji}\frac{d\gamma_i^*}{dt} = (Y9)_{ji}\alpha_i^* + (Y8)_{ji}\gamma_i^* - \frac{n_1A}{2R}E_k(Y10)_{ji}\phi_i^* \qquad (20)$$

$$K_{ji}\frac{d\delta_i^*}{dt} = E_kM_{ji}\alpha_i^* + N_{ji}\delta_i^* + a^2R(Y2)_{ji}\theta_i^* + R(Y4)_{ji}\phi_i^* \qquad (21)$$

$$\frac{n_1A}{R}C_{ji}\frac{d\theta_i^*}{dt} = A(Y2)_{ji}\delta_i^* + (Y1)_{ji}\theta_i^* + AB(Y3)_{ji}\phi_i^* \qquad (22)$$

$$\frac{n_1A}{R}P_{ji}\frac{d\phi_i^*}{dt} = \frac{n_1A}{2R}E_k(Y10)_{ji}\gamma_i^* + AV_{ji}\delta_i^* + a^2ABS_{ji}\theta_i^* + Q_{ji}\phi_i^* \qquad (23)$$

where,

$$U_{ji} = -[< DW_jDW_i > +a^2 < W_jW_i >]$$
$$E_{ji} = (1+R)[< D^2W_jD^2W_i > +2a^2 < DW_jDW_i > +a^4 < W_jW_i >] + R_e[< DW_jD^2W_i > -a^2 < W_iDW_j >]$$
$$O_{ji} = - < \Theta_jG_i >$$
$$(Y5)_{ji} = -[< DW_jDG_i > +a^2 < W_iG_j >]$$
$$(Y6)_{ji} = - < W_jDX_i >$$
$$H_{ji} = < \Theta_j\Theta_i >$$
$$I_{ji} = < [1 - f(z)]\Theta_jW_i >$$
$$J_{ji}^* = -[< D\Theta_jD\Theta_i > +a^2 < \Theta_j\Theta_i > -P_e < \Theta_iD\Theta_j >]$$
$$P_{ji} = < Z_jZ_i >$$
$$Q_{ji} = -\{< DZ_iDZ_j > +[a^2(1+AB) + 2A] < Z_jZ_i >\}$$
$$S_{ji} = < Y_iDZ_j >$$
$$V_{ji} = < X_iDZ_j >$$
$$(Y10)_{ji} = < ZjGi >$$
$$C_{ji} = < Y_iY_j >$$
$$(Y1)_{ji} = -[(1+AB) < DY_jDY_i > +(a^2 + 2A) < Y_jY_i >]$$
$$(Y3)_{ji} = - < Z_jDY_i >$$
$$(Y2)_{ji} = < X_jY_i >$$
$$N_{ji} = -[(1+R) < DX_jDX_i > +a^2(1+R) < X_jX_i >] + R_e < X_jDX_i >$$
$$K_{ji} = < X_jX_i >$$
$$M_{ji} = - < W_iDX_j >$$
$$(Y4)_{ji} = - < Z_iDX_j >$$
$$(Y7)_{ji} = < G_iG_j >$$

$(Y8)_{ji} = -[< DG_iDG_j > + (a^2 + 2A) < G_iG_j >]$
$(Y9)_{ji} = A < DG_iDG_j > +a^2A < G_jW_i >$

and $< \cdots >$ denotes integration with respect to z between the limits 0 and 1.

The set of ordinary differential equations (18)-(23) are just approximations to the system of partial differential equations (12)-(17). Following Chandrasekhar (1961), Datta and Sastry (1972) and Qin and Kaloni (1992), equations (18)-(23) must be solved for free-free, isothermal and micro-rotation free surfaces with the boundary conditions on the planes z = 0 and 1,

$$W = 0, D^2W = 0; \theta = 0; G = 0; Y = 0; DX = 0; z = 0.$$

4. Single-term Galerkin expansion technique

Taking i = j = 1, equations (18)-(23) become (dropping the subscripts for simplicity),

$$U\frac{d\alpha^*}{dt} = E\alpha^* + a^2R_aO\beta^* + R(Y5)\gamma^* + E_k(Y6)\delta^* \tag{24}$$

$$P_rH\frac{d\beta^*}{dt} = I\alpha^* + J^*\beta^* + \bar{\delta}O\gamma^* \tag{25}$$

$$\frac{n_1A}{R}(Y7)\frac{d\gamma^*}{dt} = (Y9)\alpha^* + (Y8)\gamma^* - \frac{n_1A}{2R}E_k(Y10)\phi^* \tag{26}$$

$$K\frac{d\delta^*}{dt} = E_kM\alpha^* + N\delta^* + a^2R(Y2)\theta^* + R(Y4)\phi^* \tag{27}$$

$$\frac{n_1A}{R}C\frac{d\theta^*}{dt} = A(Y2)\delta^* + (Y1)\theta^* + AB(Y3)\phi^* \tag{28}$$

$$\frac{n_1A}{R}P\frac{d\phi^*}{dt} = \frac{n_1A}{2R}E_k(Y10)\gamma^* + AV\delta^* + a^2ABS\theta^* + Q\phi^* \tag{29}$$

where,

$U = -[< (DW)^2 > +a^2 < W^2 >]$

$E = (1+R)[< (D^2W)^2 > +2a^2 < (DW)^2 > +a^4 < W^2 >] + R_e[< DWD^2W > -a^2 < WDW >]$

$O = - < \Theta G >$

$Y5 = -[< DWDG > +a^2 < WG >]$

$Y6 = - < WDX >$

$H = < \Theta^2 >$

$I = < [1 - f(z)]\Theta W >$

$J^* = -[< (D\Theta)^2 > +a^2 < \Theta^2 > -P_e < \Theta D\Theta >]$

$$P = \langle Z^2 \rangle$$
$$Q = -\{\langle (DZ)^2 \rangle + [a^2(1 + AB) + 2A] \langle Z^2 \rangle\}$$
$$S = \langle YDZ \rangle$$
$$V = \langle XDZ \rangle$$
$$Y10 = \langle ZG \rangle$$
$$C = \langle Y^2 \rangle$$
$$Y1 = -[(1 + AB) \langle (DY)^2 \rangle + (a^2 + 2A) \langle Y^2 \rangle]$$
$$Y3 = - \langle ZDY \rangle$$
$$Y2 = \langle X\,Y \rangle$$
$$N = -[(1 + R) \langle (DX)^2 \rangle + a^2(1 + R) \langle X^2 \rangle] + R_e \langle XDX \rangle$$
$$K = \langle X^2 \rangle$$
$$M = - \langle WDX \rangle$$
$$Y4 = - \langle ZDX \rangle$$
$$Y7 = \langle G^2 \rangle$$
$$Y8 = -[\langle (DG)^2 \rangle + (a^2 + 2A) \langle G^2 \rangle]$$
$$Y9 = A \langle (DG)^2 \rangle + a^2 A \langle GW \rangle$$

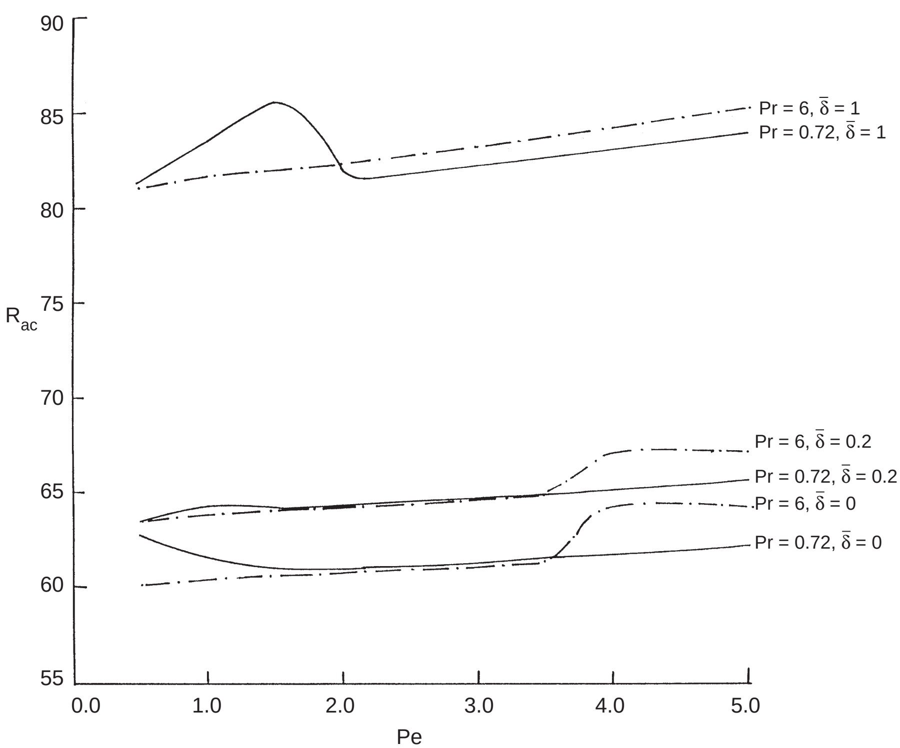

Figure 2.
Variation of R_c with P_e for different values of $\bar{\delta}$; $A = 10$, $E_k = 1$, $R = 0.1$, $J = 0.1$, $B = 0.1$

To study the stability of the system, make use of the following trial functions

$$W = z(1 - z)\left(1 + z - z^2\right); \Theta = z(1 - z); G = z(1 - z)$$
$$X = z^2(3 - 2z); Y = z(1 - z); Z = z(1 - z) \quad (30)$$

Following Finlayson (1972), equations (24)-(29) can be written in the form,

$$\frac{d\overline{A}}{dt} = L\overline{A} \quad (31)$$

where,

$$\overline{A} = [\alpha^* \beta^* \gamma^* \delta^* \theta^* \phi^*]^T$$

$$\frac{d\overline{A}}{dt} = \left[\frac{d\alpha^*}{dt}\frac{d\beta^*}{dt}\frac{d\gamma^*}{dt}\frac{d\delta^*}{dt}\frac{d\theta^*}{dt}\frac{d\phi^*}{dt}\right]^T$$

and

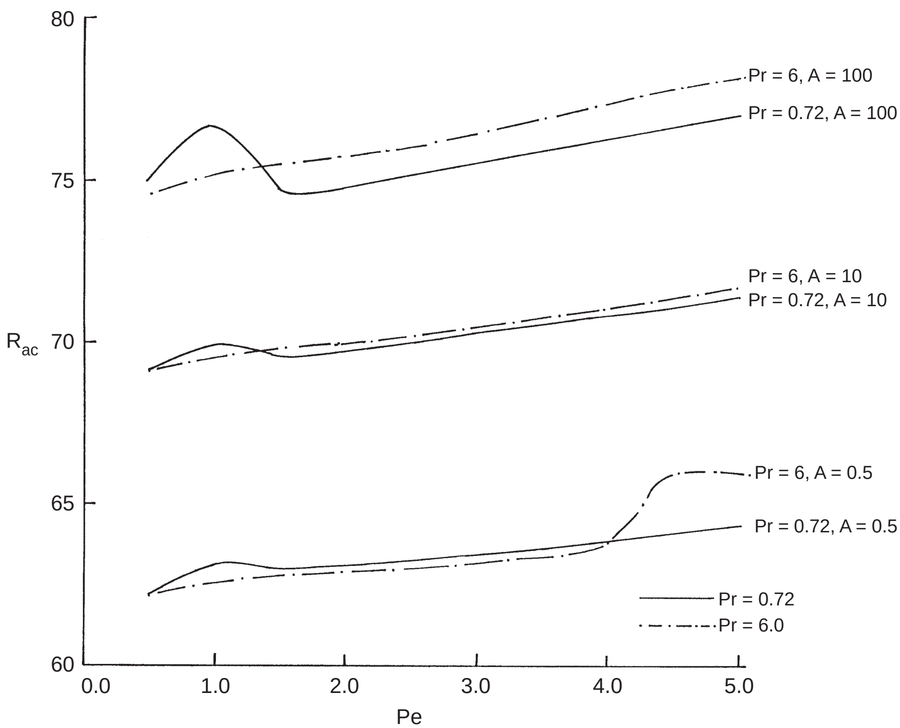

Figure 3.
Variation of R_c with P_e for different values of A; $\overline{\delta}$= 0.5, E_k = 1, R = 0.1, J = 0.1, B = 0.1

$$L = \begin{bmatrix} \frac{E}{U} & \frac{a^2R_aO}{U} & \frac{R(Y5)}{U} & \frac{E_k(Y6)}{U} & 0 & 0 \\ \frac{I}{HP_r} & \frac{J^*}{HP_r} & \frac{\bar{\delta}O}{HP_r} & 0 & 0 & 0 \\ \frac{(Y9)}{\left[\frac{n_1A(Y7)}{R}\right]} & 0 & \frac{(Y8)}{\left[\frac{n_1A(Y7)}{R}\right]} & 0 & 0 & -\frac{E_k(Y10)}{2(Y7)} \\ \frac{ME_k}{K} & 0 & 0 & \frac{N}{K} & \frac{a^2R(Y2)}{K} & \frac{R(Y4)}{K} \\ 0 & 0 & 0 & \frac{A(Y2)}{\left[\frac{n_1AC}{R}\right]} & \frac{(Y1)}{\left[\frac{n_1AC}{R}\right]} & \frac{AB(Y3)}{\left[\frac{n_1AC}{R}\right]} \\ 0 & 0 & \frac{E_k(Y10)}{2P} & \frac{AV}{\left[\frac{n_1AP}{R}\right]} & \frac{a^2ABS}{\left[\frac{n_1AP}{R}\right]} & \frac{Q}{\left[\frac{n_1AP}{R}\right]} \end{bmatrix}$$

To study the stationary instability the time derivatives in equation (31) are set to zero and thus, the stability of the system is governed by det L = 0.

Thus, the stationary Rayleigh number R_{as} is given by

$$R_{as} = \frac{J^*}{a^2O\Delta_3}\frac{\Delta_4\Delta_7 - \Delta_5\Delta_6}{I\Delta_7 - AV\bar{\delta}O\Delta_5} \qquad (32)$$

where,

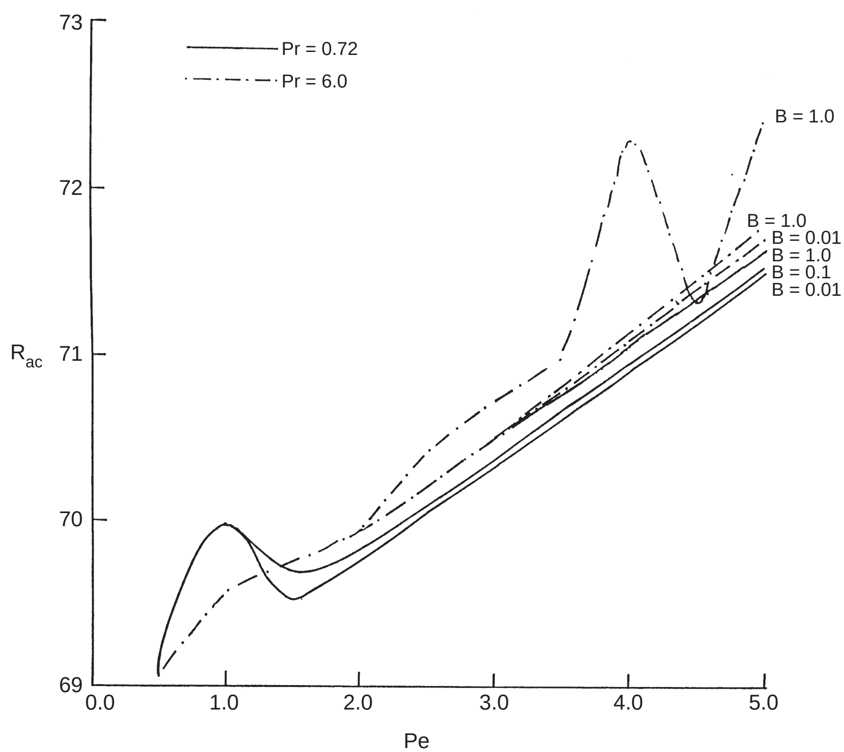

Figure 4.
Variation of R_c with P_e for different values of B; $\bar{\delta}$ = 0.5, E_k = 1, R = 0.1, J = 0.1, A = 10

$$\Delta_1 = \mathrm{RAV(Y5)} - \frac{n_1A}{2R}E_h^2\mathrm{(Y6)(Y10)}$$
$$\Delta_2 = \left[\frac{n_1A}{2R}E_k\mathrm{(Y10)}\right]\left[a^2\mathrm{AR(Y2)}^2 - \mathrm{N(Y1)}\right]$$
$$\Delta_3 = \mathrm{RAV(Y1)(Y4)} + a^2\mathrm{ARQ(Y2)}^2 - \mathrm{NQ(Y1)}$$
$$\Delta_4 = E\Delta_3 + MQE_k^2\mathrm{(Y1)(Y6)}$$
$$\Delta_5 = \mathrm{(Y9)}\Delta_3 + \frac{n_1A}{2R}E_k^2\mathrm{AVM(Y1)(Y10)}$$
$$\Delta_6 = \Delta_1\Delta_3 + E_kQ\Delta_2\mathrm{(Y6)}$$
$$\Delta_7 = \mathrm{AV}[\mathrm{(Y8)}\Delta_3 + \frac{n_1A}{2R}E_k\Delta_2\mathrm{Y10)}]$$

When $P_e \rightarrow 0$, the results coincide with those of Qin and Kaloni (1992). When $R = 0, \overline{\delta} = 0, A = 0, B = 0$, the above results coincide with those of Krishna (1997) and in addition, as $P_e \rightarrow 0$, the results coincide with those of Rudraiah and Chandana (1986).

5. Stationary convection with higher order Galerkin method

The results obtained from a single-term Galerkin method give reasonable results, but fail to take care of some of the effects arising due to throughflow. Therefore, higher order Galerkin technique is adopted in this section for stationary convection, which takes care of these effects. For this, choose suitable trial functions as follows:

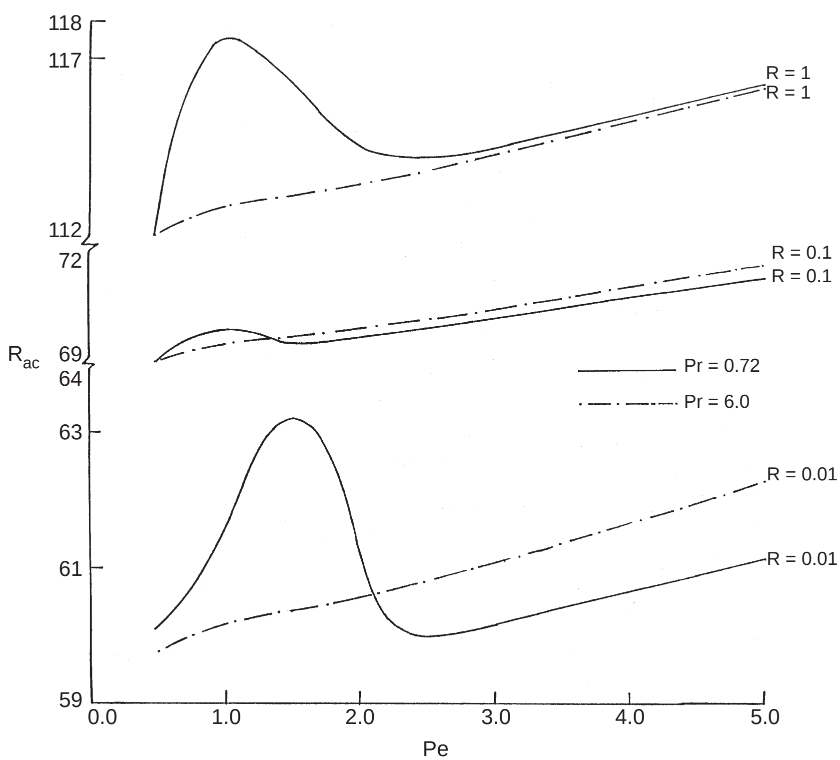

Figure 5.
Variation of R_c with P_e for different values of R; $\overline{\delta} = 0.5$, $E_k = 1$, $A = 10$, $J = 0.1$, $B = 0.1$

$$W = z(1-z)\left(1+z-z^2\right)z^{N-1}; \Theta = z(1-z)z^{N-1}; G = z(1-z)z^{N-1}$$
$$X = z^2(3-2z)z^{N-1}; Y = z(1-z)z^{N-1}; Z = z(1-z)z^{N-1} \quad (33)$$

where N is the order of the Galerkin method.

The time derivatives are set to zero to obtain the required equations for the stationary convection in equation (31). Then the set of resulting algebraic equations can have non-trivial solution if and only if, the $6N \times 6N$ determinant of the coefficients is equal to zero i.e., det $L = 0$. The above eigenvalue problem is solved initially by adopting the Newton-Raphson method of iteration for the Rayleigh number R_a by fixing all other parameters. All the integrals consisting of the elements of the above determinant are evaluated analytically, rather than numerically, to avoid errors during numerical integration. Sufficient accuracy was found for the order $N = 11$ of the Galerkin method. The critical Rayleigh numbers obtained for this approximation are depicted in the form of graphs and the results are discussed.

6. Discussion

Throughout the discussion, all the parameters occurring in the problem take values as in Qin and Kaloni (1992). Figures 2-7 show the plot of critical Rayleigh number R_{ac} versus the Péclet number P_e. From Figure 2, it is observed that for a fixed P_e, R_{ac} increases with $\bar{\delta}$. In other words, as $\bar{\delta}$

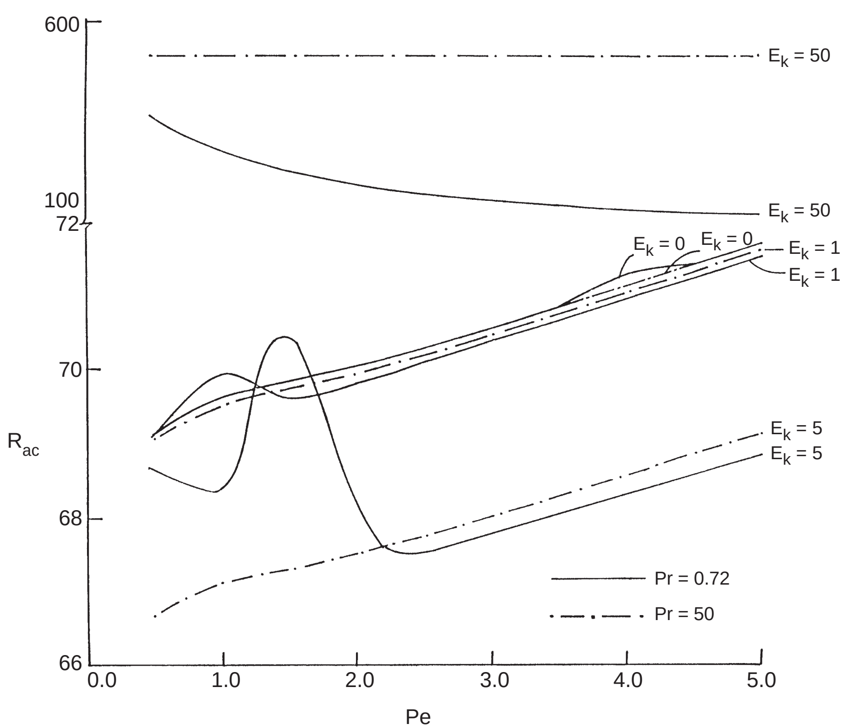

Figure 6.
Variation of R_c with P_e for different values of E_k; $\bar{\delta} = 0.5$, A = 10, R = 0.1, J = 0.1, B = 0.1

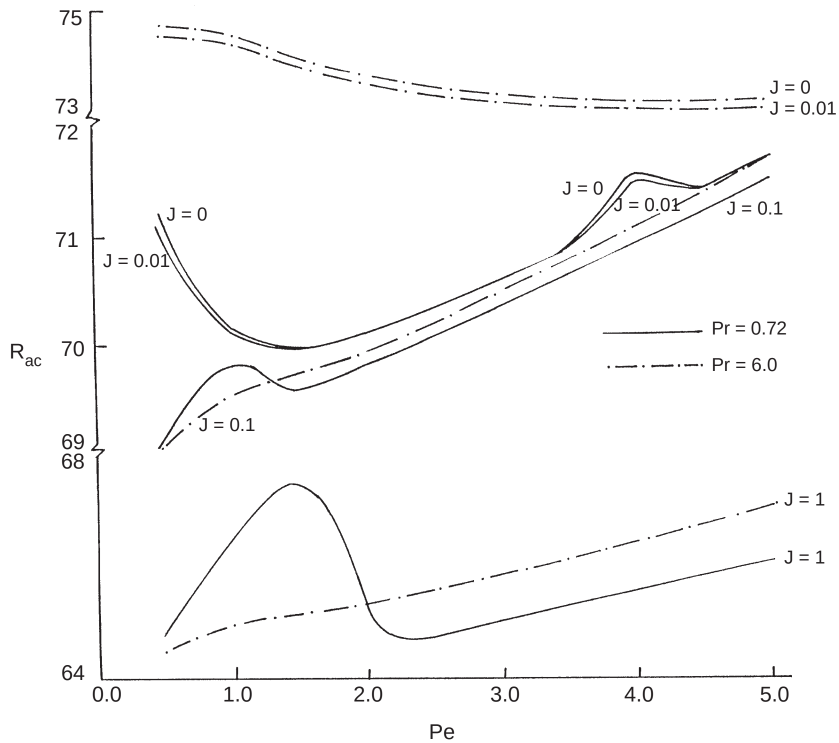

Figure 7.
Variation of R_c with P_e for different values of J; $\overline{\delta} = 0.5$, $E_k = 1$, R = 0.1, J = 0.1, B = 0.1, A = 10

increases, the heat induced into the fluid due to the micro-elements gets increased; thus reducing the heat transfer from bottom to top. The decrease in heat transfer is responsible for delaying the onset of instability. This clearly shows that the effect of an increase in $\overline{\delta}$ is to increase R_{ac}.

From Figures 3-5, it is observed that R_{ac} increases with increase in micropolar parameters A, B and R. This implies that the width of the cell at the onset of instability increases with the heat imparted by micro-rotation, while it reduces as the couple stress increases. It is noted that the onset of instability is delayed with increase in either the micro-rotation parameter R or the couple stress parameters A and B.

It is observed from Figure 6 that both stabilizing and destabilizing factors due to vertical throughflow can be enhanced by rotation. It is observed that the critical Rayleigh number decreases (i.e. destabilizes the system) up to certain values of Péclet number P_e and further increase in P_e makes the system more stable. This may be due to the distortion of basic temperature profile caused by throughflow and rotation. The distortion of the basic temperature profile leads to large values of P_e where the temperature is large and hence an increased rate of transfer of energy into the disturbance. This is in contrast to that "the effect of throughflow is invariably stabilizing for symmetric boundaries" stated by Nield (1987), Shivakumara and Jayaram (1993) and Shivakumara and Rudraiah (1997).

In the case of Newtonian fluid, rotation introduces vorticity into the fluid. Thus the fluid moves in horizontal planes with higher velocity. On account of this motion the velocity of the fluid perpendicular to the planes reduces. Thus the onset of convection is delayed. In the case of a micropolar fluid, when the system is subjected to small rotation, the micro-rotation and the rotation of the system have reinforced one another as the net effect is to curtail the vertical component of the velocity. For large rotations, the motion is reduced by the presence of micropolar additives, thereby the system is prone to instability. Finally, from Figure 7, it is observed that the micro-inertia J plays an opposite role to that played by the parameters A, B and R.

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